A Constraint Based Study of Centre Cracked Plate Specimen Using Numerical Method

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Abstract— Material fracture characterization is of paramount importance if the failure of that material leads to catastrophic damages in the machine or structure. Stress intensity factor is one such fracture parameter used to characterize the material having a crack under Linear Elastic Fracture Mechanics (LEFM) conditions. The constraints near the crack tip/front alter the value of stress components drastically. In this paper an attempt is made to study the constraints effects using various parameters stress triaxiality along the ligament and plastic zone size on CCP specimen by detailed 2D finite element analysis under LEFM conditions for various crack to width ratio. Small scale yielding was ensured by taking 2/3rd of the yield stress as applied load. Results reveal that stress triaxiality and plastic zone size act as effective secondary parameters for Stress intensity factor in LEFM to measure the level of in-plane constraint near crack tip. Authors believe that a thorough 3D analysis is necessary to understand the out of plane constraint effect.

Index Terms— Constraint issues, Finite element Analysis, Plastic zone size, Stress intensity factor, Stress triaxiality.

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1 INTRODUCTION

STRUCTURES in modern era have seen advancement due to the use of complex structural members. The complexity of the structures has been increased drastically due to the modern manufacturing processes and techniques. Due to the inevitable defects arising from the manufacturing processes such as casting, extrusions etc., cracks and certain other defects predominate. Combined with the different kinds of loads a component may also develop fatigue crack which may propagate leading to catastrophic failure of the structural member. Fracture characterization of a component has been keen area of interest from a long time. Linear Elastic Fracture Mechanics (LEFM) and Elastic Plastic Fracture Mechanics (EPFM) are the two domains which emphasize fracture characterization, while former dealing with linear analysis and the latter with nonlinear. Energy release rate (G) and Stress intensity factor (SIF, K) are the parameters that can be used for LEFM and J-integral, Crack Tip Opening Displacement (CTOD), Crack Mouth Opening Displacement (CMOD) are used for EPFM analysis. SIF is a variable combining the effect of stress (σ) and the crack length (a) to determine the state of stress at the crack tip due to applied load. Various literatures [1-2] establishes empirical relationship for stress intensity factor for different specimens. In LEFM, elastic analysis is carried out to determine stress and displacement fields near a crack tip with characterizing parameters. But most of the engineering materials do not fall in this category. The existence of the plastic zone was ignored in earlier analysis, because of practical methods were not developed to account for the elastic plastic behavior within the plastic zone.

The Plastic Zone Size (PZS) ahead of the crack tip under the remote loading depends on amount of yielding at the crack tip. If the PZS is beyond some amount of yielding then the EPFM analysis has to be employed and usage of non-linear parameters such as J-integral, CTOD etc. are essential. J integral is a line integral around the crack tip [3].Though J-integral is used for non-linear and elastic plastic analysis; it has been proved that under linear elastic analysis J-integral is equal to G [4]. Alone J-integral in many cases yield a conservative value, since it depends on the factors like a/W ratio, specimen geometry, remote loading and PZS at the crack tip. To ensure these effects in finding fracture toughness, ASTM standards like ASTM E399-17, ASTM E1820-17a, etc. have employed some restrictions on specimen geometry, specimen preparation, and conditions to be satisfied by the broken specimen after the test. Due to difficulty associated with testing of fracture toughness, many researchers [5, 6, 7] concentrated on numerical analysis of crack under various a/W, remote loading, stress triaxiality (h), etc. The effect of these parameters on fracture toughness value is significant and termed as constraint effects. Constraint literally is a structural obstacle against plastic deformation, which is induced mainly by geometrical and physical boundary conditions. The level of constraint at a crack tip plays an important role in the fracture of a cracked component and can be examined through detailed study of the crack tip stress fields.

Basically, we need a single value of fracture toughness, as it serves as a material property. To account for this, ASTM ensures the specimen must be loaded in plain strain conditions by restricting specimen geometry and making the specimen under high constraint leading to conservative value of fracture toughness. ASTM suggests the specimens should have high constraint under remote loading at the crack tip. If this fracture toughness value is used by the designers and engineers for their structural design, it leads to conservative design, which is against the design philosophy.

As long as the Plastic Zone (PZ) at the crack-tip is limited compared with the geometry of the component or specimen, so called Small Scale Yielding (SSY), a single parameter fracture mechanics approach can be applied. Single parameter characterizes the crack-tip conditions and can be used as geometry independent fracture criterion. However, the single parameter fracture mechanics breaks down as the size of the PZ due to increase in crack size, and fracture toughness will now depend on the crack size, geometry and mode of loading. In general the apparent toughness of a material changes according to the shape and size of the cracked configuration and the mode of loading imposed. Recent analytical, numerical and experimental studies have attempted to describe fracture in terms of K, J or CTOD and a second parameter. The reason for the second parameter is to provide further information, which K, J or CTOD on its own is unable to convey, concerning how the structural and loading configuration affects the constraint conditions at the crack-tip. Various second parameters used by various researchers [8, 9, 10, 11, 12, 13] are A2, Q, stress triaxiality ratio (h), PZS, T-stress to predict the level of constraint ahead of the crack tip/crack front. In general, the conditions ahead of a crack are neither plane stress nor plane strain, but are in 3-dimensional. In these situations, the tri-axial stress field near the crack-front has an important role in a fracture mechanics framework [7, 8, 14, 15]. Basically, the existing tri-axial constraints are in-plane and out-of-plane constraints (refer Fig.1) and both are related to the geometry and loading configuration of the cracked structure. The in-plane constraint is essentially dominated by the dimensions in the normal plane to the crack-front and the out-of-plane constraint is mainly determined by the dimensions parallel to the crack-front (i.e. in the thickness direction), together with the boundary condition.

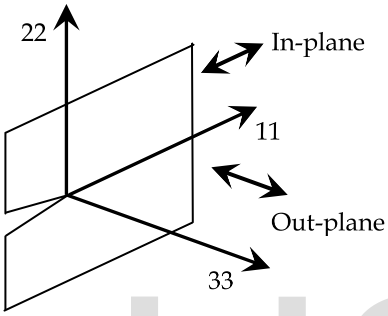

Fig. 1 The in-plane and Out-of-plane in a 3D specimen

As per authors' best knowledge and available resources, most of the researchers studied the constraint effect on fracture was on high constraint specimens, whereas work on low constraint specimen is limited. In this paper an attempt is made to study the effect of constraint parameters, viz. Triaxiality ratio (h) and PZS on low constraint specimen Center Cracked Plate (CCP) under mode-I loading for various crack to width ratio (a/W) through numerical analysis using ABAQUS software for both state of stress.

2 Finite Elelemt Analysis

A series of plane stress and plane strain FE analysis have been carried out using ABAQUS 6.14 software [16].The specimen considered for the analysis is CCP as shown in Fig.2 and the dimensions of the specimen are chosen as per Kudari et al. [17]. The width W = 25.4 mm, thickness (B) = 12.7 mm and 2H = 101.6 mm is taken by adopting H = 2W and B = 0.5W. The material considered in the present analysis is Interstitial Free (IF) steel having the Young's modulus (E) = 200 GPa and Poisson's ratio (μ) = 0.3 for the remote loading (σ) = 100 MPa.

Finite Element (FE) analysis has been carried out on one-fourth of the specimen geometry due to quarter symmetry as shown in Fig.3.The 2D analysis domain is discretized using 8-noded quadrilateral (element type CPE8R for plane strain and CPS8R for plane stress within the ABAQUS library) finite elements using reduced integration method. Similar kind of elements has been used in earlier papers [18]. A fine mesh is created surrounding the crack to get better results near the crack tip. Fig.3 shows a typical mesh along with boundary condition on the specimen. Along the ligament, displacement in y-direction is taken as zero ($u_y = 0$) and along the height, displacement in x-direction is taken as zero (ux = 0) due to quarter symmetry. Crack to width is varied between 0.1 and 0.8 with an increment of 0.05.The number of nodes are in the range of 670-6789 and maximum elements are found to be 2200.

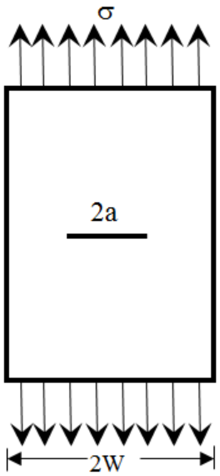

Fig.2 Centre Cracked Plate (CCP)

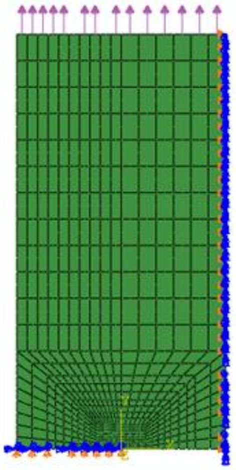

Fig.3 Typical Mesh and Boundary Condition

3 Result and Discussion

For both plane stress and plane strain condition the Stress Intensity Factor under Mode-I loading (K_I) were numerically obtained through ABAQUS. J integral method is used to extract the SIF in ABAQUS software. The maximum load on the specimen is taken as 100MPa to keep approximately in LEFM regime. The detail of extraction of K_I using ABAQUS has been reported in the work of Kodancha and Kudari [18]. The values of K_I are also calculated using analytical equation as given below [4]:

$$K_I = \sigma\sqrt{(\pi a)}Y \tag{1}$$

Where geometric factor,

$$Y = 1.0 + 0.128\left(\frac{a}{W}\right) - 0.288\left(\frac{a}{W}\right)^2 + 1.523\left(\frac{a}{W}\right)^3 \tag{2}$$

The extracted values of K_I for both state of stress and analytical values are compared for validation and plotted in Fig.4. Figure shows the excellent match between analytical and numerical results. This plot also indicates the magnitude of K_I dependency on a/W and independent of state of stress. State of stress is decided based on specimen thickness but same K_I value from the plot signifying the need of additional parameter to predict actual stress ahead of the crack tip.

Further, the variation of other stresses σ_{11}, σ_{22}, σ_{33}, $\sigma_{\text{von Mises}}$ and $\sigma_{\text{Hydrostatic}}$ along the ligament were obtained for various a/W and state of stress. A typical variation of obtained stresses for a/W= 0.55 along the ligament for both state of stress is shown in Fig.5. Figure reveals that the value of σ_{11}, σ_{22} are same for both state of stress and varies with reference to σ_{33} as expected. However von Mises and hydrostatic stresses are dependent on state of stress and are high for plane stress conditions. The analysis of fracture based on these stresses is difficult and one needs to have a parameter which combines all the stresses into account. However, hydrostatic stress be combined with von Mises stress into a single non-dimensional parameter, which characterizes a stress-state and is a measure of its triaxiality [19]. In this regard an attempt is made to study the variation of stress triaxiality (h) and is given as [20]:

$$h = \frac{(\sigma_{11} + \sigma_{22} + \sigma_{33})/3}{\sigma_{vonmises}} \tag{3}$$

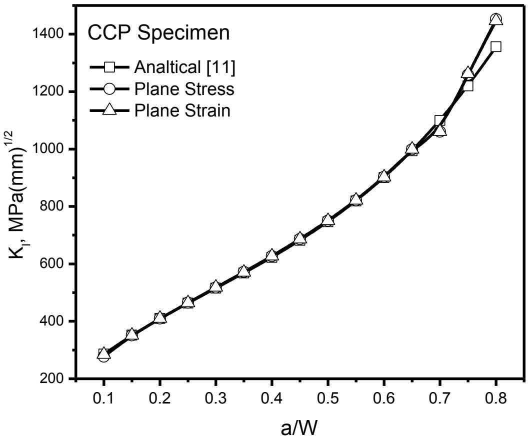

Fig.4 Variation of K_I with a/W

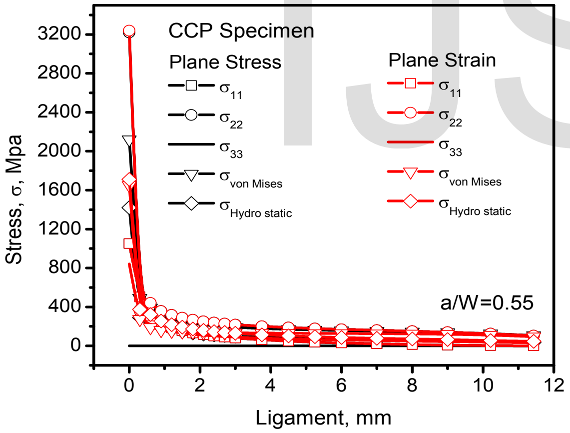

Fig.5 Variation of various stresses along the ligament

Using equation (3) h is computed for various a/W and state of stress. Fig.6 shows the variation of h along the ligament for both state of stress typically for a/W= 0.1, 0.2, 0.60, 0.70 and 0.8.

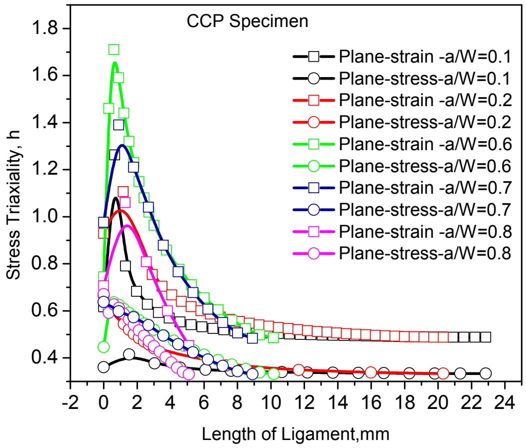

Fig.6 Variation of h along the ligament for various a/W

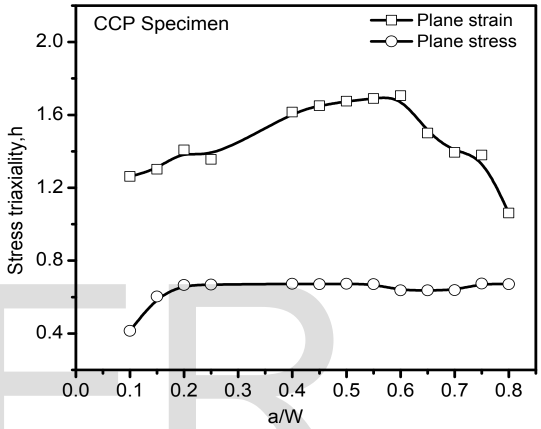

Fig.7 Variation of h for various a/W

From the figure, h is maximum in the vicinity of the crack for plane stress and plane strain conditions. Figure indicates h increases with a/W till 0.6 and then it shows a downward trend. This kind of nature may be due to the combined effect of direct and bending load at the crack tip after a/W=0.6. Figure also reveals that the value of h is high in case of plain strain condition as compared to plane stress indicating the constraint is more in case of plane strain condition. In case of plane stress condition the decrease in h is due to progressive relaxation of the stresses in the vicinity of the crack. Similar observation is found in the work of Henry and Luxmoore [21].

Peak values of h for different state of stress and a/W are shown in Fig.7. Figure reveals, for plane stress condition the variation of h is negligible after a/W=0.20, where as in plane strain condition a gradual increase in h has been observed upto a/W=0.60 and downwards afterwards. In both the figures dependency of h on state of stress and a/W is significant and can be used as a constraint parameter along with K_I to characterize the crack behavior effectively.

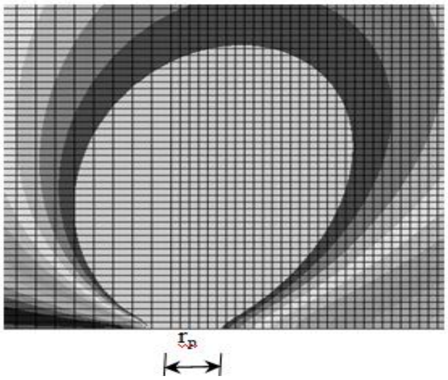

Fig.8 Method of Obtaining PZS

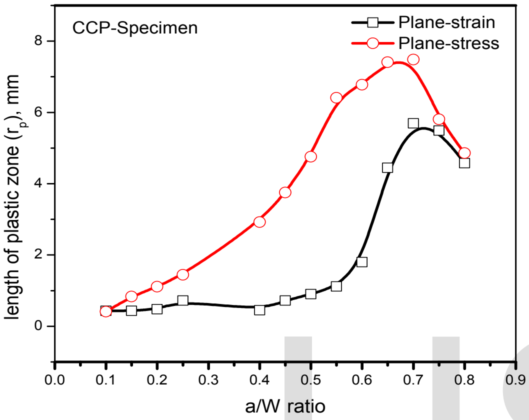

Fig.9 Length of plastic zone size for various a/W ratio

The material in the vicinity of the crack tip is most affected and deforms anelastically. From the Fig.6 it is clear that the material yields and flows to decrease the stress. According to Prashanth Kumar [4], it is clearly mentioned that the PZS may be regarded as a parameter in representing the toughness of the material. Since plastic zone plays a vital role in a fracture, an attempt has been made to investigate size of the PZ with reference to state of stress and different a/W. To understand the stress distribution around the crack tip, Irwin [22] suggested a parameter called length of plastic zone (r_p) or PZS. Earlier literature [5, 18] suggested the effect of PZS on behavior of the crack tip is very significant. In ASTM, the specimen size requirements are imposed on the basis of shape and size of the PZ. But on low constraint specimen PZS effect has to be quantified for various a/W ratio. Here an attempt is made to understand the effect of PZS (r_p) on CCP specimen for various a/W ratio under different state of stress.

The PZS is obtained from the post processor of the ABAQUS and method of obtaining is shown in Fig.8. The PZS ahead of the crack tip for various a/W ratio under both state of stress have been plotted and shown in Fig.9. Figure indicates the increase in PZS as a/W increases up to 0.7 and then followed by a downfall. This may be due to low toughness (resistance) of the specimen at higher a/W ratios and also may be development of compressive zone development from other end. In case of plane stress the PZS is more compared to plane strain condition indicating thin specimen exhibits better resistance to fracture than thick specimen. A high value of PZS indicates high resistance to the crack growth thus high strength of the specimen.

An attempt has been made to study the variation of all constraint parameter along with SIF and is plotted in Fig.10. The figure reveals the dependence of SIF, PZS and maximum value of h measured along the ligament for different state of stress and a/W. The effect of individual constraint parameter on SIF is explained earlier. For instance at a/W=0.50 , h and PZS are 1.67 and 0.9017mm for plane strain condition whereas for plane stress condition h and PZS are 0.67 and 4.699mm respectively for the same value of SIF≅747 MPa (mm)$^{1/2}$. From above it is observed that as h increases PZS decreases. At this moment it cannot be made general as it requires rigorous analysis to establish the relationships.

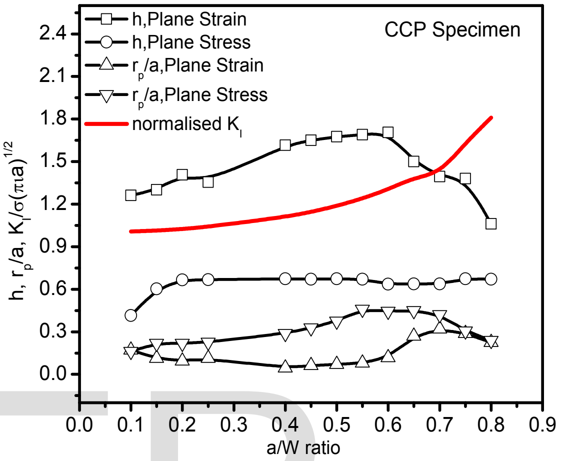

Fig.10 Variation of h, normalized rp and normalized KI for various a/W

The use of a constraint matching technique like this allows the relevant toughness to be used in a defect assessment scheme, rather than the lower bound toughness associated with low constraint specimen center cracked theoretical specimens. There are many possibilities to treat this constraint effect if one can further explore more on in plane and out of plane constraint parameters by conducting a series of 3D analysis on low constraint specimens. Experimentation of the same may be tried by providing fixture arrangements to load the specimen like one used in CT Specimen as a high constraint specimen.

4 Conclusions

The major conclusions derived from this investigation are as follows:

i. The magnitude of stress intensity factor (K_I) is independent of state of stress.
ii. The peak values of triaxiality stress (h) are same for a/W=0.2-0.8 in plane stress condition. Whereas for plane strain condition a/W=0.1–0.6 the h value increases after that downfall occurs, which reveal the dependency of h on a/W is high.
iii. Plastic Zone Size (PZS) increase with increase in a/W up to 0.7 for both state of stress. The PZS value is more in case of plane stress condition specifying thin specimen toughness is more for fracture growth.
iv. In case of h as a secondary parameter for K_I, the constraint loss is increasing as the a/W increases in case of plane

stress condition where as it is almost constant in case of plane strain up to a/W = 0.60 and almost becomes zero at a/W=0.70 and further loss of constraint is more.

v. In case of PZS as a secondary parameter for K_I, the constraint loss is constant upto a/W=0.65 for plane stress condition and then increases, however in case of plane strain condition the constraint loss is more as compared to plane stress between a/W = 0.15-0.7.

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